An approximate inverse to extended Born modeling operator

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INTRODUCTION

Inspired by ten Kroode(2012), this study derives an approximate inverse to the extended Born modeling operator in 2D. This inverse operator is valuable because the widely used imaging technique, Reverse Time Migration, is only the adjoint of modeling operator, which can position the reflector correctly but with incorrect amplitude. The derivation starts from asymptotic ray theory and uses stationary phase principle. We determine that the inverse operator differs from adjoint operator only by application of several explicit velocity-independent filters. This inverse operator, on the one hand, can be used as true amplitude migration (in asymptotic sense). On the other hand, it can be used as preconditioner to speed up the iterations of Full Waveform Inversion(FWI).

BORN MODELING OPERATOR

Constant Density Acoustic Wave Equation

$$\left(\frac{1}{v^2(\mathbf{x})}\frac{\partial^2}{\partial t^2} - \nabla^2\right)u(t,\mathbf{x};\mathbf{x_s}) = f(t,\mathbf{x};\mathbf{x_s})$$

Born Modeling Operator

$$DF[v]\delta v = \frac{\partial^2}{\partial t^2}\int d\mathbf{x}d\tau G(\mathbf{x_s},\mathbf{x},\tau)\frac{2\delta v(\mathbf{x})}{v(\mathbf{x})^3}G(\mathbf{x},\mathbf{x_r},t-\tau)$$

High Frequency Approximation of Born Modeling Operator

$$DF[v]\delta v \cong \frac{\partial}{\partial t}\int d\mathbf{x}a(\mathbf{x_s},\mathbf{x})a(\mathbf{x},\mathbf{x_r})\delta(t-T_r-T_s)\frac{2\pi\delta v(\mathbf{x})}{v(\mathbf{x})^3}$$

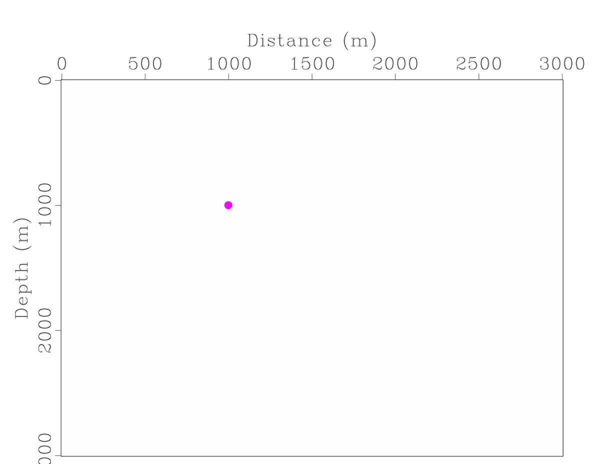
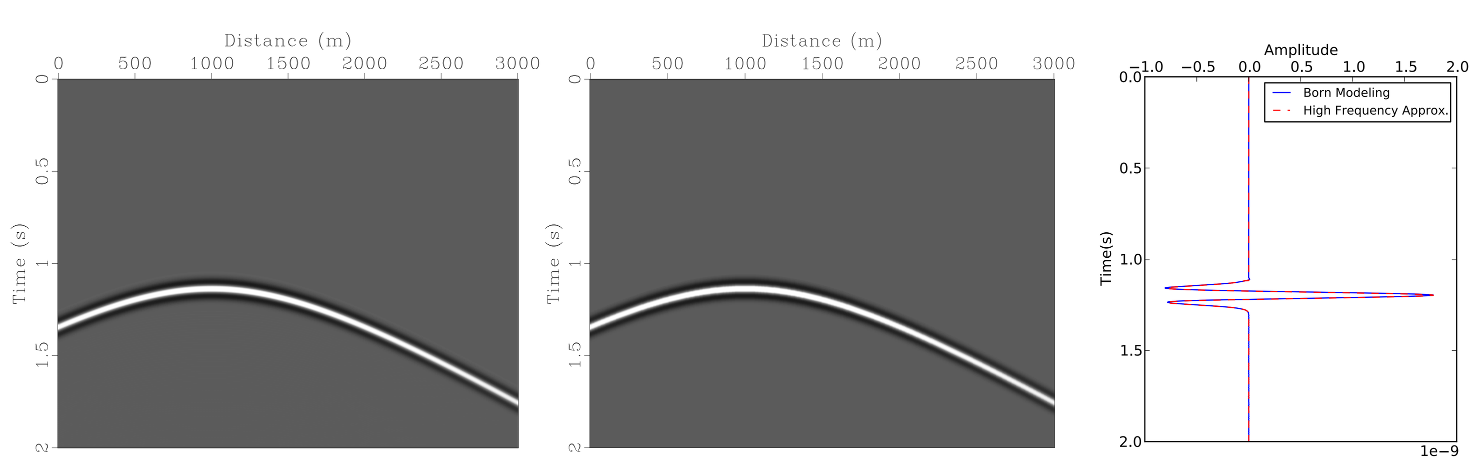


Figure 1 Impulse Response Model for Born Modeling Operator. This model is composed of a constant background velocity(2000 m/s) model and a dirac delta function perturbation model. It is designed to verify the high frequency approximation of born modeling operator in 2D.



(a) Born Modeling (b) High Frequency Approximation (c) One Trace Comparison

Figure 2 Impulse Response of the model shown in Figure 1 using (a) Born Modeling Operator (b) High Frequency Approximation. One trace Comparison is shown in (c). It illustrates the effectiveness of the high frequency approximation for Born Modeling Operator in 2D.

DEPTH-ORIENTED EXTENDED MODEL

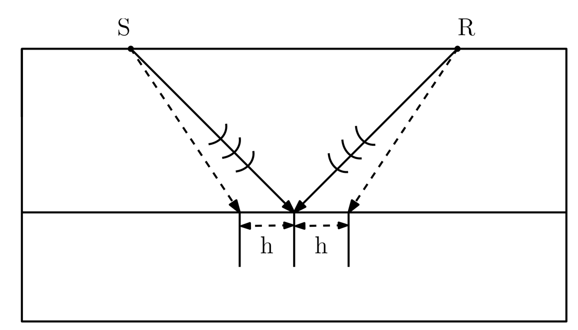


Figure 3 Schematic of horizontal subsurface offset extension. In this study, we will only consider horizontal subsurface offset h. It can be easily modified into vertical subsurface offset extension.

Extended Born Modeling Operator

$$DF[v]\delta v(\mathbf{x_s},\mathbf{x_r},t) = \frac{\partial^2}{\partial t^2}\int d\mathbf{x}d\mathbf{h}d\tau G(\mathbf{x_s},\mathbf{x}-\mathbf{h},\tau)\frac{2\delta v(\mathbf{x})}{v(\mathbf{x})^3}G(\mathbf{x}+\mathbf{h},\mathbf{x_r},t-\tau)$$

Adjoint of Extended Born Modeling Operator

$$DF^*[v] = -\frac{2}{v^3}\int d\mathbf{x_r}d\mathbf{x_s}dtd\tau G(\mathbf{x_s},\mathbf{x}-\mathbf{h},\tau)G(\mathbf{x}+\mathbf{h},\mathbf{x_r},t-\tau)\frac{\partial^2}{\partial t^2}$$

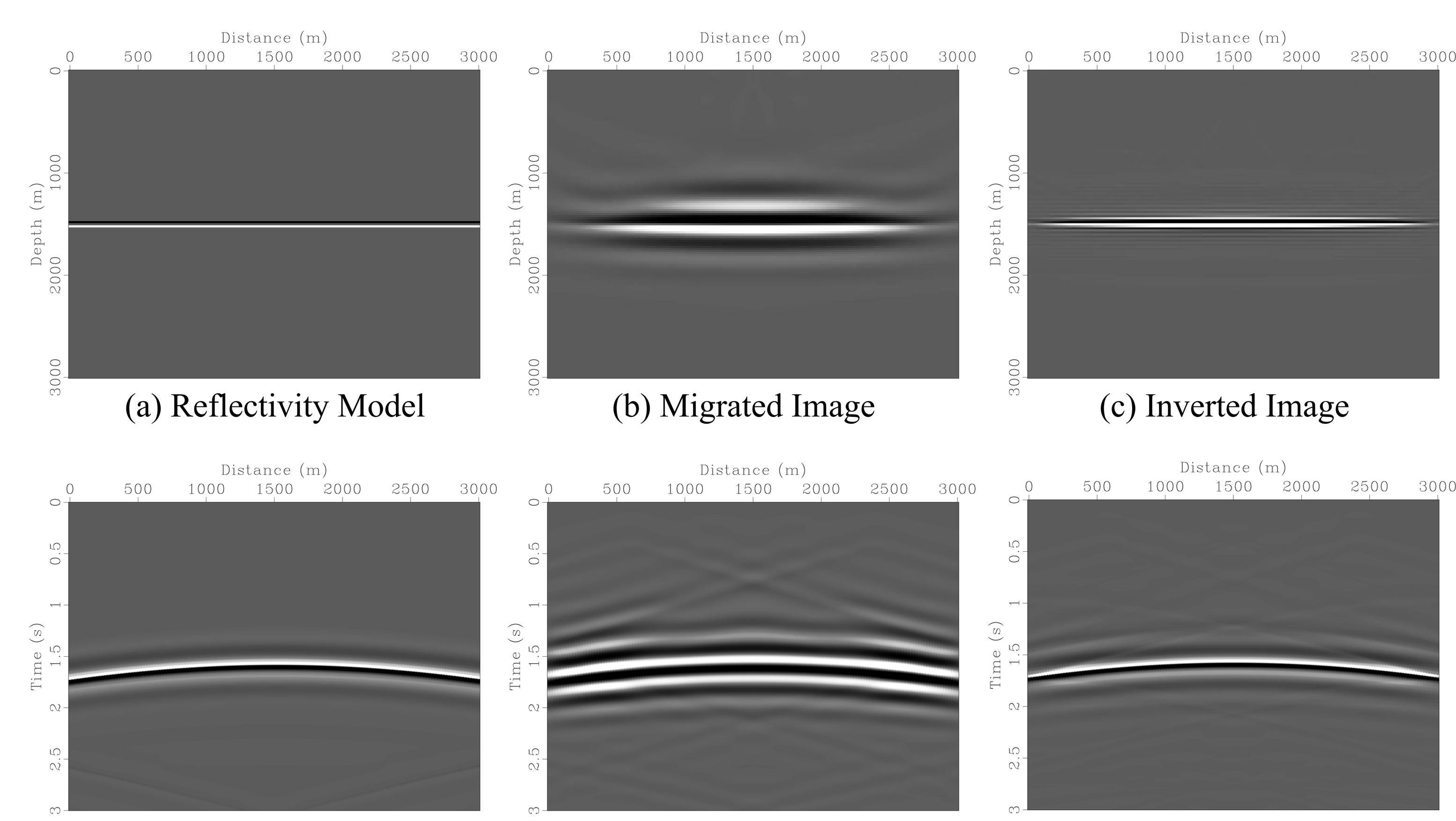
MAIN RESULT

To derive the inverse operator, we apply high frequency approximation and analyze the Normal Operator using stationary phase principle.

$$DF^{-1}[v]\delta u(\mathbf{x},h) \cong -8|kk'|v^6 DF^*[v]I_t^4 D_{z_s}D_{z_r}\delta u(\mathbf{x},h)$$

$k=(k_x,k_z)$ wavenumber
$k'=(k_h,k_z)$ extended wavenumber
I_t Time Integral
D_{z_s}, D_{z_r} Derivative with respect to z_s and z_r

NUMERICAL EXPERIMENTS



(a) Reflectivity Model (b) Migrated Image (c) Inverted Image

(d) Data of True Model (e) Data of Migrated Image (f) Data of Inverted Image

Figure 4 A simple (a) one layer reflectivity model with constant background velocity model is used to simulate (d) the Born data. Conventional Reverse Time Migration and the new Inverse Operator are respectively used to generate the (b) migrated image and (c) inverted image(only $h=0$ section displayed here). Resimulated data of both images are shown as (e) and (f).

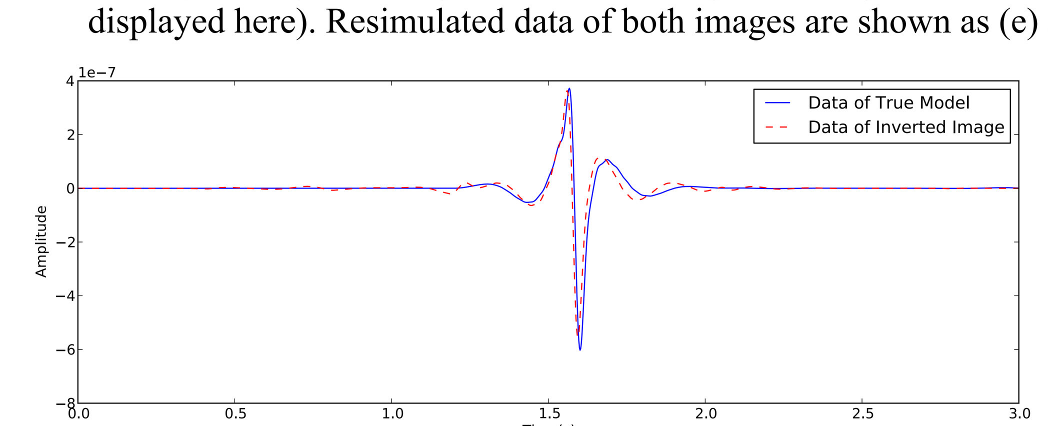


Figure 5 One trace (x=1500m) comparison of simulated data and resimulated data of inverted image.

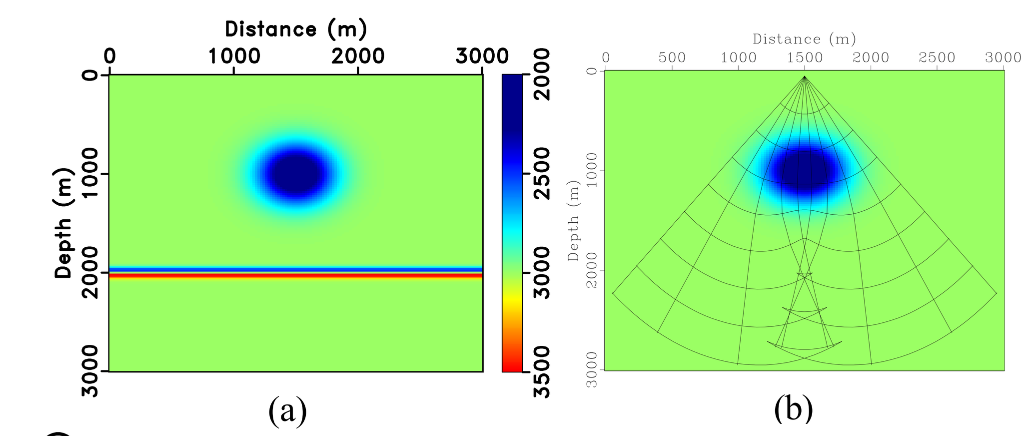


Figure 6 (a) Gaussian lens velocity model with a flat reflector at depth 2 km, which leads to multipathing (b) The background model is overlain with rays and wavefronts. A triplication occurs below the lens.

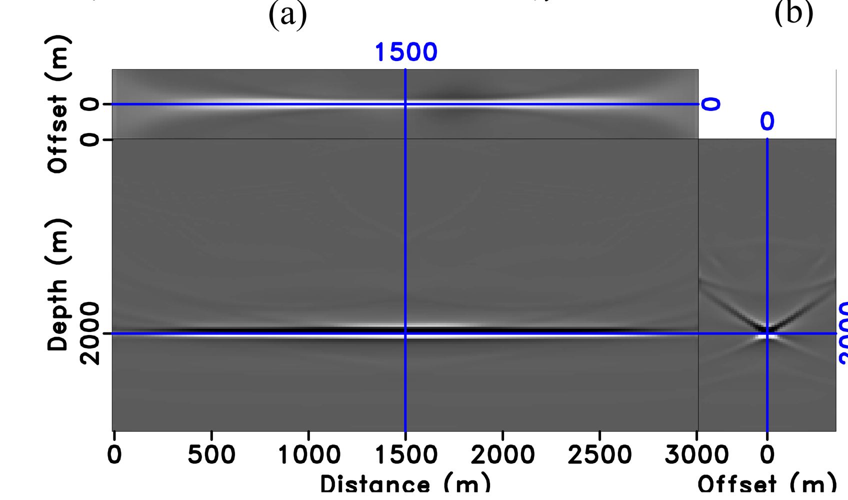
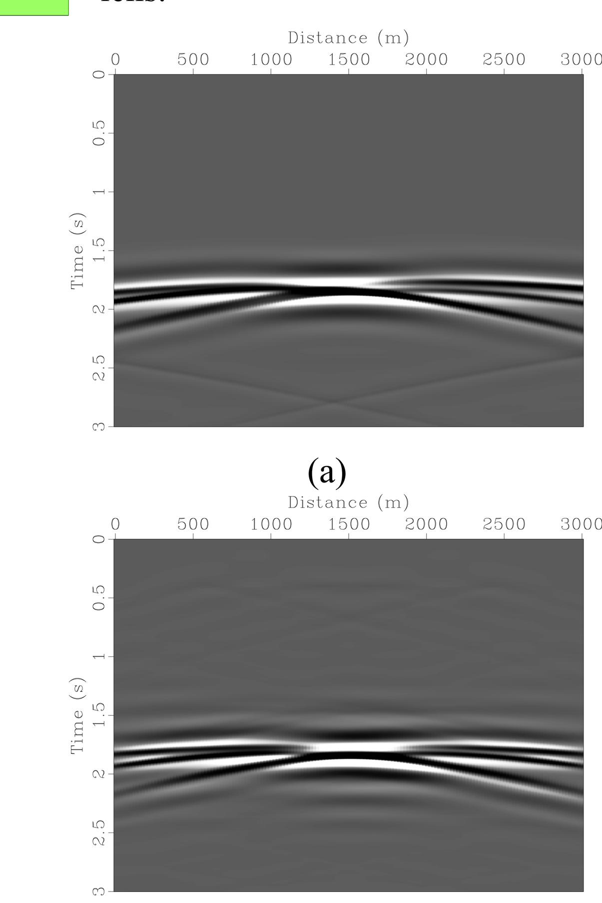


Figure 7 Inverted Image using the new operator



(a)

(b)

Figure 8 (a) Data of True Model (b) Data of Inverted Image

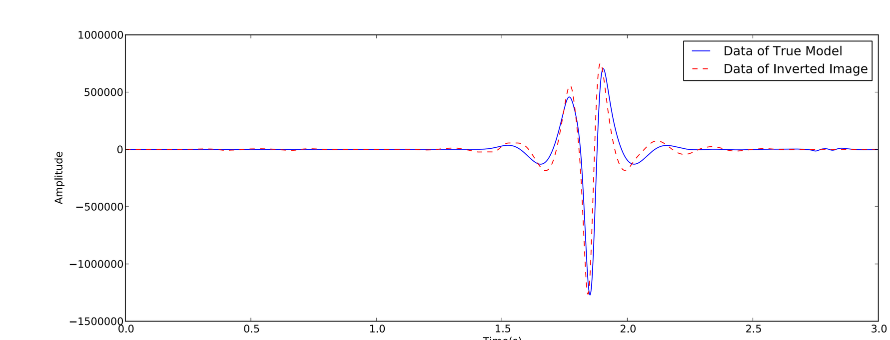


Figure 9 One trace (x=1500m) comparison of simulated data and resimulated data of inverted image

CONCLUSION

We derive an approximate inverse operator to the extended Born modeling operator. Unlike the migration, the inverse operator will reconstruct the reflection information with correct amplitude. Both theoretical proof and numerical experiments illustrate the effectiveness of the approximate inverse operator.

REFERENCES

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